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Lowest Allocation Method (LAM): A New Approach to Obtain Feasible Solution of Transportation Model

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Abstract— The study in this paper is to discuss a new approach to find the feasible solution of a Transportation Problem (TP). There are some existing algorithms to solve Transportation Problem such as North West Corner Rule (NWC), Least Cost Method (LCM), Vogel's Approximation Method (VAM) etc. where VAM is the most efficient algorithm for finding feasible solution. In this paper we proposed a new algorithm to find feasible solution of TP named "Lowest Allocation Method (LAM)" based on supply and demand. Our proposed algorithm takes minimum iteration to find feasible solution other than existing algorithms and which is close to optimal solution.

Index Terms— Transportation Problem (TP), LAM, VAM, Feasible Solution, Optimal Solution.

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1 Introduction

Transportation Problem is special types of linear programming problem where the goal is to minimize the total transportation cost. Commodities or products are shipped from sources (factories) to destinations (retail house) in such a way so that total transportation cost should be minimized. In general there are m sources and n destinations where each i^{th} source and j^{th} destinations has a capacity which are known as amount of supply S_i and amount of demand D_j respectively. Every roots ***i.e,*** source to destination has a fixed unit transportation cost C_{ij}. In our proposed method LAM, amount of commodity x_{ij} are allocated on the basis of the availability of supply and demand. The linear programming of a Transportation Model is given below:

Minimize: $$\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}x_{ij}$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} \le S_i \quad , \qquad for\ i = 1,2,3..................m$$

$$\sum_{i=1}^{m} x_{ij} \ge D_j \quad , \qquad for\ j = 1,2,3..................n$$

$$x_{ij} \ge 0, \qquad for\ all\ i, j$$

2 .

2.1 **Existing Methods:** Initially we mention that there are several algorithms exists for finding feasible solution of Transportation Problem such as North West Corner Rule (NWC), Least Cost Method (LCM), Vogel's Approximation Method (VAM) etc. Among these VAM provides the feasible solution which is nearer to optimal solution.

The existing algorithm of VAM is given below:

Step-1: Indentify the boxes having minimum and next to minimum transportation cost in each row and write the difference (Penalty) along the side of the table against the corresponding row.

Step-2: Indentify the boxes having minimum and next to minimum transportation cost in each column and write the difference (Penalty) along the side of the table against the corresponding column.

If minimum cost appear in two or more times in a row or column then select these same cost as a minimum and next to minimum cost and penalty will be zero.

Step-3: i) Indentify the row and column with the largest penalty, breaking ties arbitrarily. Allocate as much as possible to the variable with the least cost in the selected row or column. Adjust the supply and demand and cross out the satisfied row or column. If a row and column are satisfies simultaneously, only one of them is crossed out and remaining row or column is assigned a zero supply or demand.

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ii) If two or more penalty costs have same largest magnitude, then select any one of them (or select most top row or extreme left column).

Step-4:

i) If exactly one row or one column with zero supply or demand remains uncrossed out, Stop.

ii) If only one row or column with positive supply or demand remains uncrossed out, determine the basic variables in the row or column by the Least-Cost Method.

iii) If all uncrossed out rows or column have (remaining) zero supply or demand, determined the zero basic variables by the Least-Cost Method. Stop.

iv) Otherwise, go to Step-1.

2.2 Proposed Algorithm for Lowest Allocation Method (LAM):

Method (LAM): The proposed method developed by us in this investigation seems to be easier than other methods and takes minimum iteration to reach feasible solution. The algorithm of LAM is given below:

Step-1: $if\ S_i < 0\ and\ D_j < 0$, then stop.

Step-2:

$$if \sum_i S_i > \sum_j D_j \quad or \quad \sum_i S_i < \sum_j D_j$$

then balance the transportation problem adding dummy demand or supply.

Step-3: Identify the smallest amount between supply and demand $i.e, \min(S_i, D_j)$. Allocate this smallest amount x_{ij} in the lowest cost cell in that row or column which contains this smallest supply or demand.

Step-3: If the smallest amount between supply and demand is appear in two or more rows or columns then select that row or column which has the lowest cost among them.

Step-4: Adjust the supply and demand and cross out the satisfied row or column. If row and column are satisfied simultaneously then crossed out one of them and set zero supply or demand in remaining row or column.

Step-5: If exactly one row or one column with zero supply or demand remains uncrossed out, Stop. Otherwise go to Step-3.

Numerical Illustration: Consider some transportation problem and solve these by Lowest Allocation Method (LAM) and compare results with the other existing methods.

Example-1:
Consider a Mathematical Model of a Transportation Problem in Table-1.1:

Source	Destinations				Supply
	D1	**D2**	**D3**	**D4**	
S1	7	5	9	11	**30**
S2	4	3	8	6	**25**
S3	3	8	10	5	**20**
S4	2	6	7	3	**15**
Demand	**30**	**30**	**20**	**10**	

Table-1.1

Solution of Example-1 using proposed method Lowest Allocation Method (LAM):

Iteration-1:

Source	Destinations				Supply
	D1	**D2**	**D3**	**D4**	
S1	7	5	9	11	**30**
S2	4	3	8	6	**25**
S3	3	8	10	5	**20**
S4	2	6	7	3 **10**	**5**
Demand	**30**	**30**	**20**		

Here destination D4 has the smallest value of demand among the all supply and demand. Allocate this amount in the lowest cost cell $(S4, D4)$ in column D4. Adjust supply and demand amount and crossed out the D4 column.

Iteration-2:

Source	Destinations				Supply
	D1	**D2**	**D3**	**D4**	
S1	7	5	9	11	**30**
S2	4	3	8	6	**25**
S3	3	8	10	5	**20**
S4	2 **5**	6	7	3 **10**	
Demand	**25**	**30**	**20**		

Here source S4 has the smallest value of demand among the all supply and demand. Allocate this amount in the lowest cost cell $(S4, D1)$ in row S4. Adjust supply and demand amount and crossed out the S4 row.

Iteration-3:

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Source	Destinations				Supply
	D1	D2	D3	D4	
S1	7	5	9	11	30
S2	4	3	8	6	25
S3	3 20	8	10	5	
S4	2 5	6	7	3 10	
Demand	5	30	20		

Iteration-4:

Source	Destinations				Supply
	D1	D2	D3	D4	
S1	7	5	9	11	30
S2	4 5	3	8	6	20
S3	3 20	8	10	5	
S4	2 5	6	7	3 10	
Demand		30	20		

Iteration-5:

Source	Destinations				Supply
	D1	D2	D3	D4	
S1	7	5	9	11	30
S2	4 5	3 20	8	6	
S3	3 20	8	10	5	
S4	2 5	6	7	3 10	
Demand		10	20		

Iteration-6:

Source	Destinations				Supply
	D1	D2	D3	D4	
S1	7	5 10	9 20	11	
S2	4 5	3 20	8	6	
S3	3 20	8	10	5	
S4	2 5	6	7	3 10	
Demand			0		

Final feasible solution Table:

Source	Destinations				Supply
	D1	D2	D3	D4	
S1	7	5 10	9 20	11	30
S2	4 5	3 20	8	6	25
S3	3 20	8	10	5	20
S4	2 5	6	7	3 10	15
Demand	30	30	20	10	

Table-1.2

Total Transportation Cost:

$$(5\times10)+(9\times20)+(4\times5)+(3\times20)+(3\times20)+(2\times5)$$
$$+(3\times10)=410$$

Solution of Example-1 using Vogel's Approximation Method (VAM):

Source	Destinations				Supply
	D1	D2	D3	D4	
S1	7	5 30	9	11	30
S2	4 25	3 0	8	6	25
S3	3 5	8	10 5	5 10	20
S4	2	6	7 15	3	15
Demand	30	30	20	10	

Total Transportation Cost:

$$(5\times30)+(4\times25)+(3\times0)+(3\times5)+(10\times5)+(5\times10)$$
$$+(7\times15)=470$$

Optimal Solution: The Optimal Transportation Cost of Example-1 is: 410

Observation: We observed that in above Example-1 Lowest Allocation Method (LAM) gives the feasible solution is 410 which is lower than Vogel's Approximation Method (VAM) where the feasible solution is 470. And also the feasible solution of LAM is exactly equal to the optimal solution.

Example-2:

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Consider another Mathematical Model of a Transportation Problem in Table-2.1:

Source	Destinations				Supply
	D1	D2	D3	D4	
S1	19	30	50	12	**7**
S2	70	30	40	60	**10**
S3	40	10	60	20	**18**
Demand	**5**	**8**	**7**	**15**	

Table-2.1

Solution of Example-2 using proposed method Lowest Allocation Method (LAM):

Source	Destinations				Supply
	D1	D2	D3	D4	
S1	19 **5**	30	50	12 **2**	**7**
S2	70	30 **3**	40 **7**	60	**10**
S3	40	10 **5**	60	20 **13**	**18**
Demand	**5**	**8**	**7**	**15**	

Total Transportation Cost:

$$(19\times5)+(12\times2)+(30\times3)+(40\times7)+(10\times5)+(20\times13)=799$$

Solution of Example-2 using Vogel's Approximation Method (VAM):

Source	Destinations				Supply
	D1	D2	D3	D4	
S1	19 **5**	30	50	12 **2**	**7**
S2	70	30	40 **7**	60 **3**	**10**
S3	40	10 **8**	60	20 **10**	**18**
Demand	**5**	**8**	**7**	**15**	

Total Transportation Cost:

$$(19\times5)+(12\times2)+(40\times7)+(60\times3)+(10\times8)+(20\times10)=859$$

Optimal Solution: The Optimal Transportation Cost of Example-1 is: 799

Observation: We observed that in above Example-2 Lowest Allocation Method (LAM) gives the feasible solution is 799 which is lower than Vogel's Approximation Method (VAM) where the feasible solution is 859. And also the feasible solution of LAM is exactly equal to the optimal solution.

Result Analysis: In above two examples we observed that Lowest Allocation Method (LAM) provides the better solution than other methods. We also check several examples other than above examples that most of the times LAM provides feasible solution which are lower than or equal to the existing methods and sometimes LAM gives optimal solution directly. The comparison table of these solutions is given below:

Transportation problem	Optimal Solution	Methods			
		LAM	VAM	LCM	NWC
Example-1	**410**	**410**	470	435	540
Example-1	**799**	**799**	859	894	975

Conclusion: In this paper we developed a new algorithm of Transportation Model named "Lowest Allocation Method (LAM)", most of the time which provides the feasible solution lower than other methods and close to or sometimes equal to the optimal solution. This proposed method is easier than other methods to compute feasible solution. But it has some limitations that's why sometimes LAM provides higher feasible solution than VAM. For logical computational procedure of LAM, it takes minimum iteration to obtain feasible solution.

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